Question # 01:

Rectangular co-ordinates of a point are $\left(3,3,-3\sqrt{3}\right)$.

a. Convert Rectangular co-ordinates to Spherical co-ordinates
b. Convert Rectangular co-ordinates to Cylindrical co-ordinates.
c. Verify your answer by converting back Rectangular co-ordinates from any one of these, that is, either from Spherical co-ordinates or Cylindrical co-ordinates.

Solution:

Rectangular co-ordinates (x, y, z) to Spherical co-ordinates (ρ, θ, ϕ)

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{45}$$

$$\tan\theta = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$\cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}} \Rightarrow \phi = Cos^{-1}\left(\frac{-3\sqrt{3}}{\sqrt{45}}\right) = Cos^{-1}\left(\frac{-\sqrt{3}}{\sqrt{5}}\right)$$

$$\left(\sqrt{45}, \frac{\pi}{4}, Cos^{-1}\left(\frac{-\sqrt{3}}{\sqrt{5}}\right)\right)$$

Rectangular co-ordinates (x, y, z) to Cylindrical co-ordinates (r, θ, z)

$$r = \sqrt{x^2 + y^2} = \sqrt{18}$$

$$\tan\theta = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$z = -3\sqrt{3}$$

$$\left(\sqrt{18}, \frac{\pi}{4}, -3\sqrt{3}\right)$$

Spherical co-ordinates (ρ, θ, ϕ) to Rectangular co-ordinates (x, y, z)

$$x = \rho\, Sin\phi\, Cos\,\theta = \sqrt{45}.\frac{\sqrt{2}}{\sqrt{5}}.\frac{1}{\sqrt{2}} = 3$$

$$y = \rho\ Sin\phi\, Sin\,\theta = \sqrt{45}.\frac{\sqrt{2}}{\sqrt{5}}.\frac{1}{\sqrt{2}} = 3$$

$$z = \rho\, Cos\,\phi = \sqrt{45}\,.\frac{-\sqrt{3}}{\sqrt{5}} = -3\sqrt{3}$$

$$\left(3, 3, -3\sqrt{3}\right)$$

Cylindrical co-ordinates (r, θ, z) to Rectangular co-ordinates (x, y, z)

$$x = r\cos\theta = \sqrt{18}.Cos\frac{\pi}{4} = \sqrt{18}.\frac{1}{\sqrt{2}} = \sqrt{9} = 3$$

$$y = r\sin\theta = \sqrt{18}\,.\frac{1}{\sqrt{2}} = 3$$

$$z = z = -3\sqrt{3}$$

Rectangular Co-ordinates	Spherical Co-ordinates	Cylindrical Co-ordinates
$\left(3,3,-3\sqrt{3}\right)$	$\left(\sqrt{45}, Cos^{-1}(\frac{-\sqrt{3}}{\sqrt{5}}), \frac{\pi}{4}\right)$	$\left(\sqrt{18}, \frac{\pi}{4}, -3\sqrt{3}\right)$

Question # 02:

Describe the set of all points in xyz-coordinate system at which *f* is continuous.

$$f(x,y,z)=\sqrt{10-x}\quad y\ln z$$

Solution:

$$f(x,y,z)=\sqrt{10-x}\quad y\ln z$$

$$D=\{(x,y,z): x\le 10, z>0\}$$

Question # 03:

By considering different path approach, find whether

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{3x^2+2y^2}$$ exist or not.

(Note: In order to get full marks, do all necessary steps)

Solution:

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{3x^2+2y^2}$$

$put\ y = mx$

$$= \lim_{(x,mx)\to(0,0)} \frac{x^2}{3x^2+2m^2x^2}$$

$$= \lim_{(x,mx)\to(0,0)} \frac{x^2}{x^2(3+2m^2)}$$

$$= \frac{1}{3+2m^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{3x^2+2y^2}$$

$put\ y = x$

$$= \lim_{x\to 0} \frac{x^2}{5x^2} = \frac{1}{5}$$

Since along different paths we have different limits, hence the limit does not exist.

Question # 01 Marks = 10

Use chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \phi}$ if $w = x^2 + y^2 + z^2$ where

$$x = r\cos\theta\sin\phi$$
$$y = r\cos\theta\cos\phi$$
$$z = r\sin\theta$$

Solution:

To find $\frac{\partial w}{\partial r}$, using chain rule, we have

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

Substituting the values, we get

$$\frac{\partial w}{\partial r} = (2x)(\cos\theta\sin\phi) + (2y)(\cos\theta\cos\phi) + (2z)(\sin\theta)$$

Putting the values of x, y and z, it yields

$$\begin{aligned}\frac{\partial w}{\partial r} &= 2r\cos^2\theta\sin^2\phi + 2r\cos^2\theta\cos^2\phi + 2r\sin^2\theta \\ &= 2r\left[\cos^2\theta(\sin^2\phi + \cos^2\phi) + \sin^2\theta\right] \\ &= 2r\left(\cos^2\theta + \sin^2\theta\right) \\ &= 2r\end{aligned}$$

Similarly, for $\frac{\partial w}{\partial \phi}$, using chain rule, we get

$$\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \phi} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \phi}$$

Substituting the values, it becomes

$$\frac{\partial w}{\partial \phi} = (2x)(r\cos\theta\cos\phi) - (2y)(r\cos\theta\sin\phi) + (2z)(0)$$

Putting the values of x and y, it yields

$$\frac{\partial w}{\partial \phi} = 2r^2\cos^2\theta\sin\phi\cos\phi - 2r^2\cos^2\theta\cos\phi\sin\phi$$

$$= 0$$

Question # 02 Marks = 10

Find the equation of tangent plane to the surface $z = \ln\sqrt{x^2 + y^2}$ at the point ***P***(-1,0,0) .

Solution:

The given surface is $z = \ln\sqrt{x^2 + y^2}$ which can be written as

$$f(x, y, z) = \ln\sqrt{x^2 + y^2} - z = 0 ------(1)$$

Taking partial derivatives of eq.(1) with respect to x, y and z, we get

$$f_x = \frac{x}{x^2 + y^2}$$
$$f_y = \frac{y}{x^2 + y^2}$$
$$f_z = -1$$

At P(-1,0,0), It becomes

$$f_x(-1,0,0) = -1$$
$$f_y(-1,0,0) = 0$$
$$f_z(-1,0,0) = -1$$

The general form of equation of tangent plane is

$$f_x(x_0, y_0, z_0)(x - x_0) + f_y(x_0, y_0, z_0)(y - y_0) + f_z(x_0, y_0, z_0)(z - z_0) = 0$$

which becomes

$$-(x+1) + 0(y-0) - 1(z-0) = 0$$
$$x + z + 1 = 0$$

which is the required equation of tangent plane.

Question # 03 Marks = 05

Find the critical points of the function given as

$f(x, y) = x^3 - 3xy - y^3$

Solution:

The given function is $f(x, y) = x^3 - 3xy - y^3$

Taking partial derivatives of the given function with respect to x and y, we get

$$f_x = 3x^2 - 3y$$
$$f_y = -3x - 3y^2$$

For the critical points, put $f_x = f_y = 0$ which yields

$$x^2 - y = 0 \Rightarrow x^2 = y \quad ---- (1)$$
$$-x - y^2 = 0 \Rightarrow x = -y^2 \quad ----- (2)$$

Substituting eq. (2) in eq. (1), we get

$$y^4 - y = 0 \Rightarrow y(y^3 - 1) = 0 \Rightarrow y = 0,\ y = 1$$

Substituting these values of y in eq. (2), we get

$$x = 0,\ x = -1.$$

Thus, the required critical points are $(0,0)$ *and* $(-1,1)$

(Note: In order to get full marks, do all necessary steps)

Question No: 1

If $f(x,y) = x^2 + 2y^2 - x^2 y$, find the local extrema and saddle points of f .

Solution:

$f(x,y) = x^2 + 2y^2 - x^2 y$

$f_x = 2x - 2xy$

$f_y = 4y - x^2$

For critical points

$f_x = 0 = 2x - 2xy \Rightarrow 2x(1-y) = 0 \Rightarrow x = 0 \text{ or } y = 1$

$f_y = 0 = 4y - x^2 \Rightarrow x^2 = 4y$ ------------------(1)

Using x=0 in (1) we have y=0

(0,0) is critical point.

Using y=1 in (1) we have

$x^2 = 4 \Rightarrow x = \pm 2$

(2,1) and (-2,1) are critical points.

Now we check on these three critical points for extremum.

$f_{xx} = 2 - 2y$

$f_{yy} = 4$

$f_{xy} = -2x$

$D = f_{xx}.f_{yy} - [f_{xy}]^2 = (2-2y)(4) - [-2x]^2$

For (0,0) $D = f_{xx}.f_{yy} - [f_{xy}]^2 = 2(4) - 0 = 8 > 0$

$f_{xx} = 2 > 0$

So f is minimum at (0,0).

For (2,1) $D = f_{xx}.f_{yy} - [f_{xy}]^2 = (0)(4) - [-4]^2 = -16 < 0$

So (2,1) is a saddle point.

For (-2,1) $D = f_{xx}.f_{yy} - [f_{xy}]^2 = (0)(4) - [4]^2 = -16 < 0$

So (-2,1) is a saddle point.

Question No: 2

Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x+y)dydx$;a>0

Solution:

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} (x+y)dydx$$

$$=\int_0^a (xy+\frac{y^2}{2})|_0^{\sqrt{a^2-x^2}} \text{ dx}$$

$$= \int_0^a (x\sqrt{a^2-x^2}+\frac{a^2-x^2}{2}) \text{ dx}$$

$$=\frac{1}{-2} \int_0^a -2x\sqrt{a^2-x^2}dx + \frac{1}{2}\int_0^a (a^2-x^2)\text{dx}$$

$$= \frac{1}{-2}\frac{(a^2-x^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}|_0^a + \frac{1}{2} (a^2x-\frac{x^3}{3})|_0^a$$

$$= \frac{1}{-2}\frac{(a^2-a^2)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{-2}\frac{(a^2-0)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} (a^3-\frac{a^3}{3})$$

$$=0 + \frac{a^3}{3} + \frac{a^3}{3} = \frac{2a^3}{3}$$

Question No: 3

Evaluate $\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^2} \sin\frac{y}{x} dydx$

Solution:

$$\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^2} \sin\frac{y}{x}\, dydx$$

$$=\int_{\sqrt{\pi}}^{\sqrt{2\pi}} -\frac{(\cos\frac{y}{x})}{\frac{1}{x}}\Big|_0^{x^2} \text{ dx}$$

$$= \int_{\sqrt{\pi}}^{\sqrt{2\pi}} (x - x\cos x) \text{ dx}$$

$$= \int_{\sqrt{\pi}}^{\sqrt{2\pi}} xdx \text{ - } \int_{\sqrt{\pi}}^{\sqrt{2\pi}} (x\cos x)dx$$

$$=\frac{x^2}{2}\Big|_{\sqrt{\pi}}^{\sqrt{2\pi}} \text{ - } [\text{xsinx}|_{\sqrt{\pi}}^{\sqrt{2\pi}} \text{ - } \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \text{sinx dx}]$$

$$= \frac{2\pi}{2} \text{- } \frac{\pi}{2} \text{ - [xsinx}|_{\sqrt{\pi}}^{\sqrt{2\pi}} \text{ - (-cosx)}|_{\sqrt{\pi}}^{\sqrt{2\pi}} \text{]}$$

$$=\frac{2\pi}{2} \text{- } \frac{\pi}{2} \text{ - [}\sqrt{2\pi} \text{ sin}\sqrt{2\pi} \text{ - } \sqrt{\pi} \text{ sin}\sqrt{\pi} \text{ +cos}\sqrt{2\pi}\text{-cos}\sqrt{\pi} \text{]}$$

$$=\frac{\pi}{2} - \sqrt{2\pi} \text{ sin}\sqrt{2\pi} + \sqrt{\pi} \text{ sin}\sqrt{\pi} \text{ - cos}\sqrt{2\pi} \text{ +cos}\sqrt{\pi}$$

(Note: In order to get full marks, do all necessary steps)

Question No: 1 **Marks =10**

Evaluate the following integral by converting it to polar coordinates.

$$\iint_R \frac{1}{1+x^2+y^2}dA$$

where R is the sector in the first quadrant bounded by $y=0$ and $y=x$ and $x^2+y^2=4$.

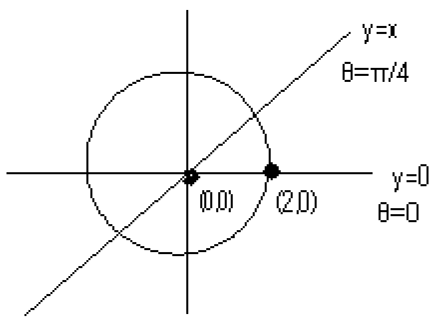
Solution:

Given integral is

$$\iint_R \frac{1}{1+x^2+y^2}dA$$

For conversion to polar coordinates, we have

$$x^2+y^2=r^2 \Rightarrow \frac{1}{1+x^2+y^2}=\frac{1}{1+r^2}$$



r ranges from 0 to 2

θ ranges from 0 to $\frac{\pi}{4}$

$dA = rdrd\theta$

Thus, the integral in polar coordinates becomes

$$\int_0^{\pi/4}\int_0^2 \frac{1}{1+r^2}rdrd\theta$$

Multiplying and dividing by 2

$$\frac{1}{2}\int_{0}^{\pi/4}\int_{0}^{2}\frac{2r}{1+r^2}drd\theta$$

$$=\frac{1}{2}\int_{0}^{\pi/4}\left[\ln(1+r^2)\right]_0^2 d\theta$$ [By the rule of integration $\int \frac{f^{'}(x)}{f(x)}dx = \ln f(x)$]

$$=\frac{1}{2}\int_{0}^{\pi/4}(\ln 5-\ln 1)d\theta$$

$$=\frac{\ln 5}{2}\int_{0}^{\pi/4}d\theta$$

$$=\frac{\ln 5}{2}[\theta]_0^{\pi/4}$$

$$=\frac{\ln 5}{2}[\frac{\pi}{4}-0]$$

$$=\frac{\pi\ln 5}{8}$$

which is the required answer.

Question No: 2 Marks =10

Find the arc length of the curve

$$\vec{r}(t)=(\cos^3 t)\hat{i}+(\sin^3 t)\hat{j}+2\hat{k} \qquad 0\le t\le \pi/2$$

Solution:

The given position vector is

$$\vec{r}(t)=(\cos^3 t)\hat{i}+(\sin^3 t)\hat{j}+2\hat{k} \text{ where } 0\le t\le \pi/2$$

In parametric form,

$$x = \cos^3 t, \quad y = \sin^3 t, \quad z = 2$$
$$\Rightarrow \frac{dx}{dt} = -3\cos^2 t \sin t, \quad \frac{dy}{dt} = 3\sin^2 t \cos t, \quad \frac{dz}{dt} = 0$$

The arc length of the curve is

$$s = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$
$$= \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t + 0}\, dt$$
$$= \int_0^{\pi/2} \sqrt{9\cos^2 \sin^2 t(\cos^2 t + \sin^2 t)}\, dt$$
$$= 3\int_0^{\pi/2} \cos t \sin t dt$$
$$= \left[\frac{3\sin^2 t}{2}\right]_0^{\pi/2}$$
$$= \frac{3}{2}\left[\sin^2 \frac{\pi}{2} - \sin^2 0\right]$$
$$= \frac{3}{2}$$

which is the required answer.

(Note: In order to get full marks, do all necessary steps)

Question # 1:

Evaluate the line integral

$\int_c (ydx - x^2 dy)$, where C is the curve x=t, y=$\frac{1}{2}t^2$ $\quad where \; 0 \le t \le 2.$

Solution:

$$\int_c (ydx - x^2dy)$$

$$= \int_0^2 ydx - x^2dy$$

$$= \int_0^2 [\frac{1}{2}t^2 - t^2.t]dt$$

$$= \frac{1}{2}\int_0^2 t^2dt - \int_0^2 t^3dt$$

$$= \frac{1}{2}.\frac{t^3}{3}|_0^2 - \frac{t^4}{4}|_0^2$$

$$= \frac{4}{3} - 4$$

$$= -\frac{8}{3}$$

Question # 2:

Use Green's theorem to evaluate the integral $\oint_c ((x^2 - y)dx + xdy)$, where C is the circle

$x^2 + y^2 = 4$ and c is oriented counterclockwise.

Solution:

$$\oint_c ((x^2 - y)dx + xdy)$$

Using Green's Theorem.

$$\oint_c \text{Pdx+Qdy} = -\oiint_R (\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x})dxdy$$

$$\frac{\partial P}{\partial y} = -1, \frac{\partial Q}{\partial x} = 1$$

$$-\oiint_R (-1-1)dA$$

$$= 2\oiint_R \ dA$$

$$= 2\int_0^{2\pi}\int_0^{2} rdrd\theta$$

$$= 4\int_0^{2\pi} d\theta$$

$$= 8\pi$$

Question # 3:

If $A = (x^4 - y^2z^2)i + (x^2 + y^2)j - x^3y^3z^3k$, determine curl of A at the point (1,4,-3).

Solution:

$$CurlA = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^4 - y^2z^2 & x^2 + y^2 & -x^3y^3z^3 \end{vmatrix}$$

$$CurlA = -3x^3y^2z^3 \ \text{i} + (3x^2y^3z^3 - 2y^2z) \ \text{j} + (\text{2x+2yz}^2) \ \text{k}$$
$$CurlA = 1296i - 5088j + 74k \quad \text{at } (1,4,-3)$$

Question No: 1 Marks =15

Determine a half range cosine series to represent the function given by

$$f(t) = 2t + 3 \quad 0 < t < 2$$
$$f(t) = f(t+4)$$

Solution:

To obtain a cosine series, i.e., a series with no sine term involved, we need an even function. Therefore, we assume the wave form to be symmetric about y-axis.

Now, to find the expressions for the Fourier coefficient, we have

$$a_{\circ} = \frac{4}{T}\int_{0}^{T/2} f(t)dt \quad \text{where T=4 is period of the given function.}$$
$$= \frac{4}{4}\int_{0}^{4/2} (2t+3)dt$$
$$= \left|\frac{2t^2}{2} + 3t\right|_0^2$$
$$= \left\{(2)^2 + 3(2)\right\} - \left\{(0)^2 + 3(0)\right\}$$
$$= 4 + 6 + 0$$
$$= 10$$

$$a_n = \frac{4}{T}\int_{0}^{T/2} f(t)\cos n\omega t dt$$
$$= \frac{4}{4}\int_{0}^{4/2} (2t+3)\cos n\omega t dt$$

Integrating by parts, we get

$$a_n = \left|(2t+3)\frac{\sin n\omega t}{n\omega}\right|_0^2 - \int_0^2 2\frac{\sin n\omega t}{n\omega}dt$$

$$= \left|(2t+3)\frac{\sin n\omega t}{n\omega}\right|_0^2 - \frac{2}{n\omega}\left|\frac{(-\cos n\omega t)}{n\omega}\right|_0^2$$

$$= \frac{7\sin n\omega 2}{n\omega} + \frac{2}{n^2\omega^2}[\cos n\omega 2 - 1]$$

Since, $\omega = \frac{2\pi}{T} = \frac{\pi}{2}$

Therefore,

$$a_n = \frac{7\sin n\pi}{n\omega} + \frac{2}{n^2\omega^2}[\cos n\pi - 1]$$

$\sin n\pi = 0,$

$$\cos n\pi = \begin{cases} 1 & \text{(n is even)} \\ -1 & \text{(n is odd)} \end{cases}$$

So,

$$a_n = \begin{cases} 0 & \text{(n is even)} \\ \frac{-4}{n^2\omega^2} & \text{(n is odd)} \end{cases}$$

Thus,

$$f(t) = \frac{a_\circ}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t$$

$$= 5 - \frac{4}{\omega^2}\left\{\cos\omega t + \frac{1}{9}\cos 3\omega t + \frac{1}{25}\dots\right\}$$

which is the required fourier series.

Question No: 2 **Marks =15**

Verify Stokes' theorem for a hemisphere S defined as $x^2 + y^2 + z^2 = 9 \quad (z \geq 0)$ where a vector field $F = z^2\hat{i} + 2x\hat{j} - y\hat{k}$ exists over the surface and around its boundary c.

Solution:

Stokes' theorem states that

$$\int_S CurlF.ds = \oint_c F.dr$$

(a)

$$\oint_c F.dr = \oint_c (z^2\hat{i} + 2x\hat{j} - y\hat{k}).(\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$= \oint_c z^2dx + 2xdy - ydz$$

For converting it to polar coordinates,

$x = r\cos\theta, \ \ y = r\sin\theta, \ \ z = 0$

where r=3 is the radius of the sphere.

$x = 3\cos\theta, \ \ y = 3\sin\theta, \ \ z = 0$

$dx = -3\sin\theta d\theta, \ \ dy = 3\cos\theta d\theta, \ \ dz = 0$

$0 \leq \theta \leq 2\pi$

Thus, the above integral becomes,

$$\int_0^{2\pi} 2(3\cos\theta)(3\cos\theta d\theta)$$

$$= 18\int_0^{2\pi} \cos^2\theta d\theta$$

$$= 18\int_0^{2\pi} \frac{1+\cos 2\theta}{2} d\theta$$

$$= 9\left[\int_0^{2\pi} d\theta + \int_0^{2\pi} \cos 2\theta d\theta\right]$$

$$= 9[\theta]_0^{2\pi} + 9\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= 9(2\pi - 0) + \frac{9}{2}(\sin 4\pi - \sin 0)$$

$$\oint_c F.dr = 18\pi \quad \text{Eq.(A)}$$

(b)

$$curlF = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & 2x & -y \end{vmatrix}$$

$$= -\hat{i} + 2z\hat{j} + 2\hat{k}$$

$$\int_s curlF.d\vec{s} = \int_s curlF.\hat{n}ds$$

where

$$\hat{n} = \frac{\nabla s}{|\nabla s|} = \frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{\sqrt{4x^2 + 4y^2 + 4z^2}}$$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{3}$$

So,

$$\int_s curlF.d\vec{s} = \int_s (-\hat{i} + 2z\hat{j} + 2\hat{k}).(\frac{x\hat{i} + y\hat{j} + z\hat{k}}{3})ds$$

$$= \frac{1}{3}\int_s (-x + 2yz + 2z)ds$$

For converting it to spherical coordinates, we have

$$x = 3\sin\phi\cos\theta$$
$$y = 3\sin\phi\sin\theta$$
$$z = 3\cos\phi$$
$$ds = 9\sin\phi d\phi d\theta$$

So, the above integral becomes,

$$\int_s curlF.d\vec{s}$$
$$= \frac{1}{3}\int_s \{-3\sin\cos\theta + 2(3\sin\phi\sin\theta)(3\cos\phi) + 6\cos\phi\} 9\sin\phi d\phi d\theta$$
$$= 3\int_s \{-3\sin^2\phi\cos\theta + 18\sin^2\phi\sin\theta\cos\phi + 6\cos\phi\sin\phi\} d\phi d\theta$$
$$= 3\int_0^{2\pi}\int_0^{\pi/2} -3\sin^2\phi\cos\theta d\phi d\theta + 3\int_0^{2\pi}\int_0^{\pi/2} 18\sin^2\phi\sin\theta\cos\phi d\phi d\theta + 3\int_0^{2\pi}\int_0^{\pi/2} 6\cos\phi\sin\phi d\phi d\theta$$
$$= I_1 + I_2 + I_3 \quad \text{Eq.(1)}$$

$$I_1 = -9\int_0^{2\pi}\int_0^{\pi/2} \sin^2\phi\cos\theta d\phi d\theta$$
$$= -9\int_0^{2\pi}\int_0^{\pi/2} \frac{1-\cos 2\phi}{2}\cos\theta d\phi d\theta$$
$$= -9\int_0^{2\pi}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]_0^{\pi/2}\cos\theta d\theta$$
$$= -9\int_0^{2\pi}\left[\frac{\pi}{4} - \frac{\sin\pi}{4}\right]\cos\theta d\theta$$
$$= \frac{-9\pi}{4}\int_0^{2\pi}\cos\theta d\theta$$
$$= \frac{-9\pi}{4}\left[\sin\theta\right]_0^{2\pi}$$
$$= \frac{-9\pi}{4}(0)$$
$$I_1 = 0 \quad \text{Eq.(2)}$$

$$I_2 = 54\int_0^{2\pi}\int_0^{\pi/2} \sin^2\phi \sin\theta \cos\phi d\phi d\theta$$

$$= 54\int_0^{2\pi}\left[\frac{\sin^3\phi}{3}\right]_0^{\pi/2} \sin\theta d\theta$$

$$= \frac{54}{3}\int_0^{2\pi}\left[\sin^3\frac{\pi}{2} - \sin 0\right]\sin\theta d\theta$$

$$= 18\int_0^{2\pi} \sin\theta d\theta$$

$$= -18\left[\cos\theta\right]_0^{2\pi}$$

$$= -18\left[\cos 2\pi - \cos 0\right]$$

$$= -18(1-1) = 0 \quad \text{Eq.(3)}$$

$$I_3 = 18\int_0^{2\pi}\int_0^{\pi/2} \cos\phi \sin\phi d\phi d\theta$$

$$= 18\int_0^{2\pi}\left[\frac{\sin^2\phi}{2}\right]_0^{\pi/2} d\theta$$

$$= 9\int_0^{2\pi} (1-0) d\theta$$

$$= 9\left[\theta\right]_0^{2\pi}$$

$$= 18\pi \quad \text{Eq.(4)}$$

Substituting the values of eqs.(2), (3), (4) in eq. (1), we get

$$\int_s curlF.d\vec{s} = 18\pi \quad \text{Eq.(B)}$$

Hence, from the equality of Eq. (A) and (B), Stokes' theorem is verified.

(Note: In order to get full marks, do all necessary steps)